LECTURE: 3-2 THE PRODUCT AND QUOTIENT RULES

Example 1: How do we find the derivative of a product? Is it true that $(fg)' = f'g'$? Why or why not?

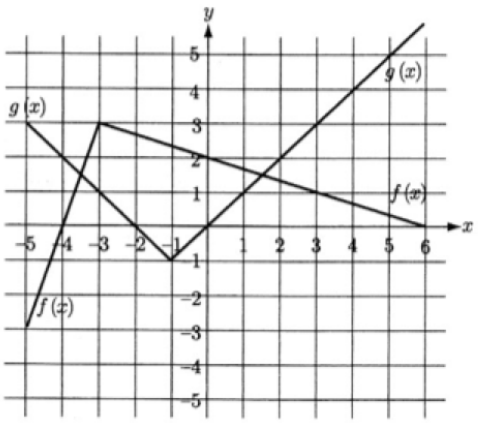
The Product Rule: If f and g are differentiable then,

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

Proof of why this is true:

Example 2: If $f(x) = xe^x$ find $f'(x)$. Then find the second and third derivatives to find a formula for the nth derivative $f^{(n)}(x)$.

Example 3: If $h(x) = f(x)g(x)$ as shown below, find $h'(3)$.



The Quotient Rule If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

Example 4: Find y' when $y = \dfrac{x^2 + x - 2}{x^3 + 6}$

Example 5: Suppose that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$ and $g'(5) = 2$. Find the following values.

(a) $(f - g)'(5)$ (b) $(fg)'(5)$ (c) $(g/f)'(5)$

Note: Don't use the product or quotient rule unless you have to. Here are a few examples where you can use the product or quotient rules, but it's easier not to!

Example 6: Find the derivative of the following functions.

(a) $f(t) = \sqrt{t}(2t + 5)$

(b) $y = \dfrac{t^3 + t + 5}{t^4}$

Example 7: Find the derivatives of the following functions

(a) $f(z) = (z^2 - \sqrt{z})(z^2 + \sqrt{z})$

(b) $y = \dfrac{\sqrt{x} - 1}{\sqrt{x} + 1}$

Example 8: Find the derivative of $f(x) = \dfrac{xe^x}{x + e^x}$.

Example 9: Find an equation of the tangent line and normal line to the given curve $y = 2\sqrt{x}e^x + 1$ at the point $(0, 1)$.

Example 10: A manufacturer produces socks. The quantity q of these socks (measured in pairs of socks) that are sold are a function of the selling price p (in dollars), so we can write $q = f(p)$. Then the total revenue earned with a selling price p is $R(p) = pf(p)$.

(a) What does it mean to say $f(10) = 20,000$ and $f'(10) = 3,500$?

(b) Assuming the values in part (a), find $R'(10)$ and interpret your answer.